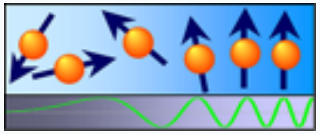
Experimental Physics EP2a

Electricity and Thermodynamics

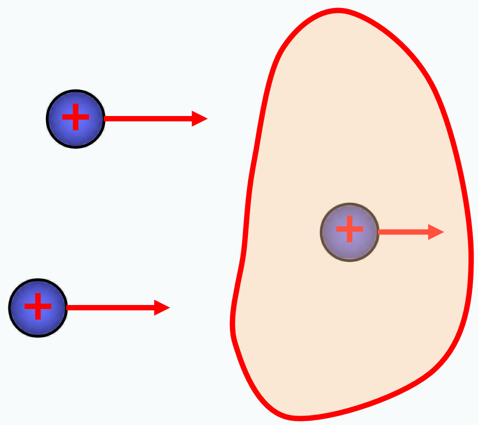
– Electric current –

Ohm‘s law, power, circuits, Kirhoff‘s rules



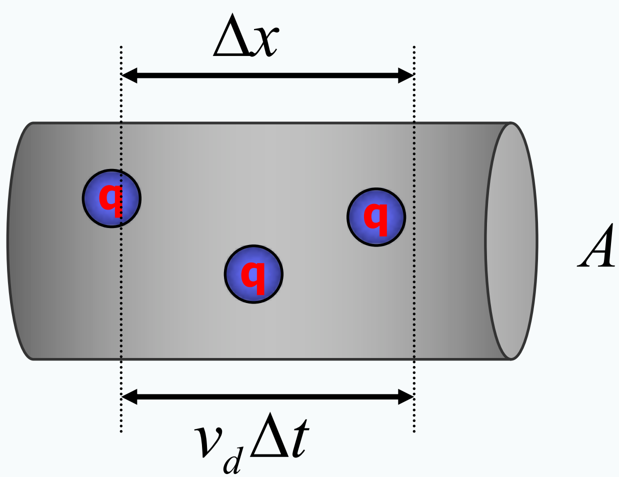
https://bloch.physgeo.uni-leipzig.de/amr/

Electric current



$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad I = \frac{dQ}{dt} \qquad \left[\frac{C}{s}\right] = [A]$$

The direction of current is conventionally assumed to be opposite to the direction of flow of electrons.



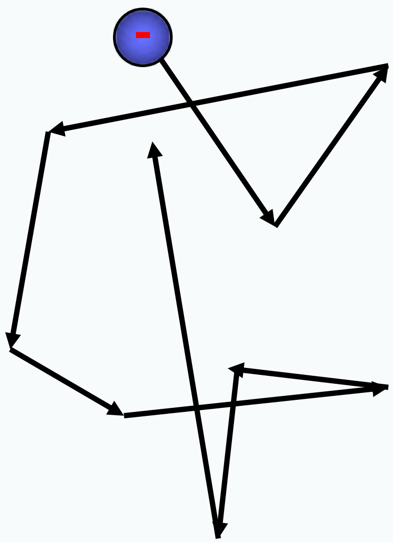
$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{nqAv_d\Delta t}{\Delta t} = nqAv_d$$

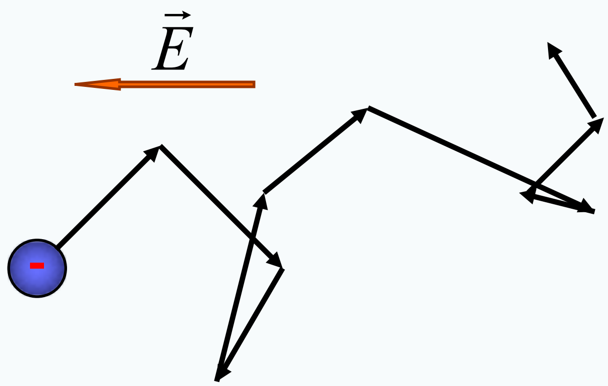
$$A = 3\times10^{-6}\ \mathrm{m}^2 \qquad \rho_c = 8.95\ \mathrm{g/cm}^3$$

$$M_c = 63.5\ \mathrm{g/mol} \qquad I = 10\ \mathrm{A}$$

$$v_d \approx 2\times10^{-4}\ \mathrm{m/s}$$

Drude's theory of electric conduction





$$\vec{F} = q\vec{E} \quad = m_e\vec{a}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t \quad = \vec{v}_i + \frac{q\vec{E}}{m}t$$

$$\vec{v}_d = \frac{q\vec{E}}{m}\tau$$

average time between collisions

$$\vec{J} = nq\vec{v}_d \qquad \vec{J} = \sigma\vec{E}$$

$$\boxed{\vec{J} = nq^2\frac{\vec{E}}{m_e}\tau}$$

$$\tau = \frac{l}{\bar{v}}$$

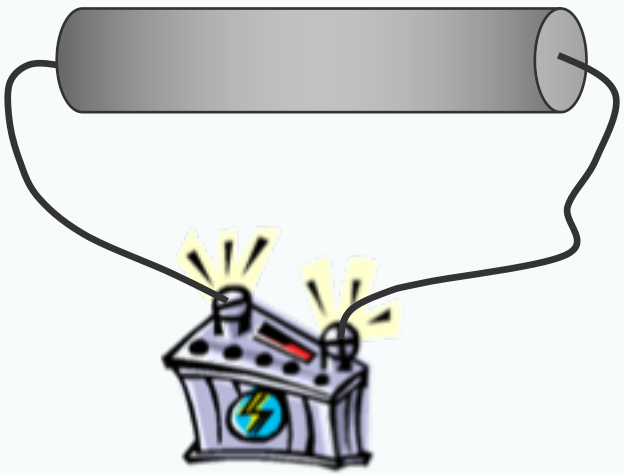
Ohm's law

$$\vec{J} \equiv \frac{\vec{I}}{A} = nq\vec{v}_d$$

$$\boxed{\vec{J} = \sigma\vec{E}}$$

σ - conductivity

Materials obeying this equation are *ohmic*.



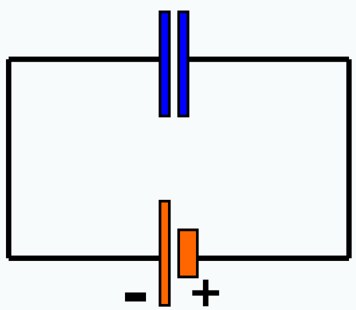
$$\Delta V = E \cdot l \quad = \frac{J \cdot l}{\sigma} \quad = \frac{l}{A\sigma} I$$

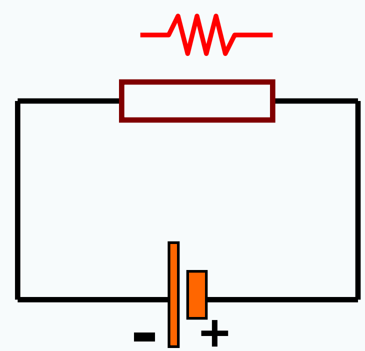
$$R \equiv \frac{l}{A\sigma}$$

resistance $[\Omega]$

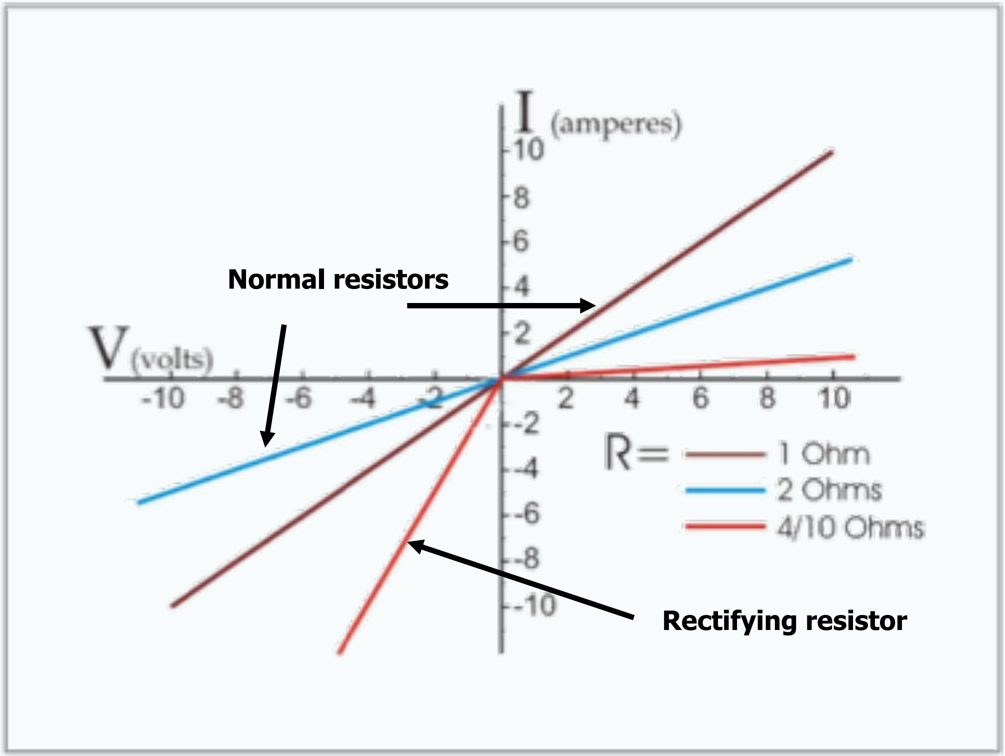
$$\rho \equiv \frac{1}{\sigma} = R\frac{A}{l}$$

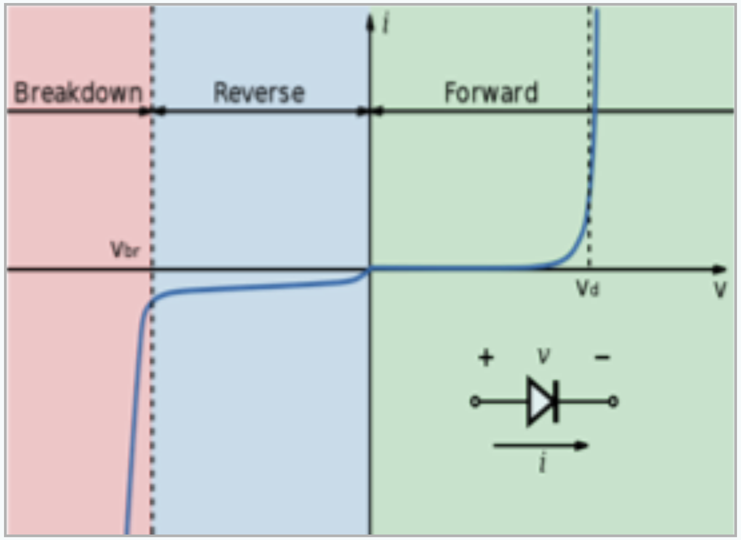
resistivity

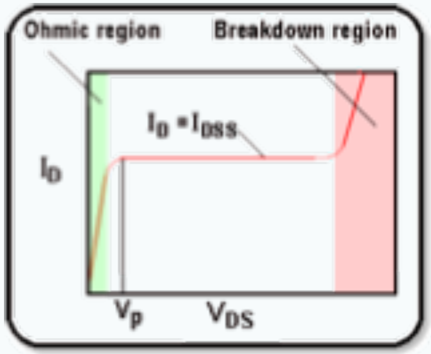


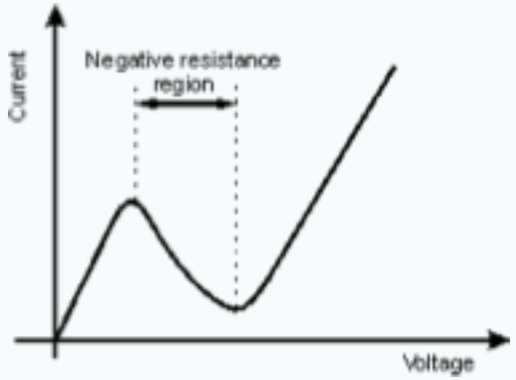


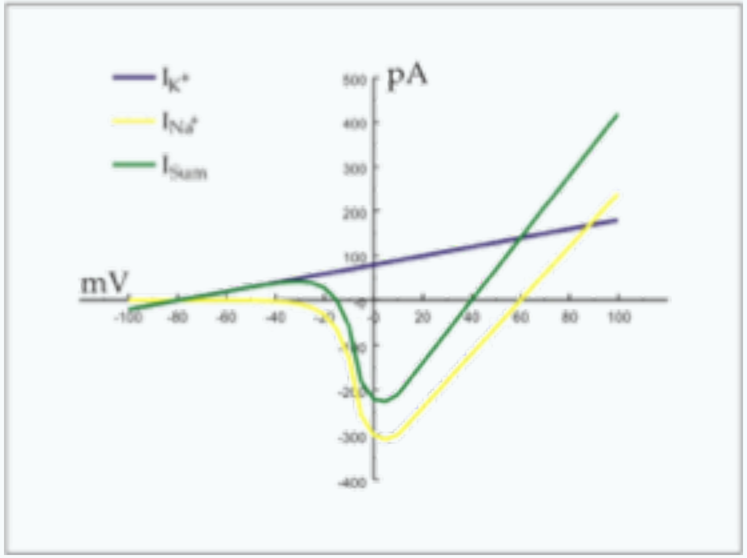
Current-potential curve



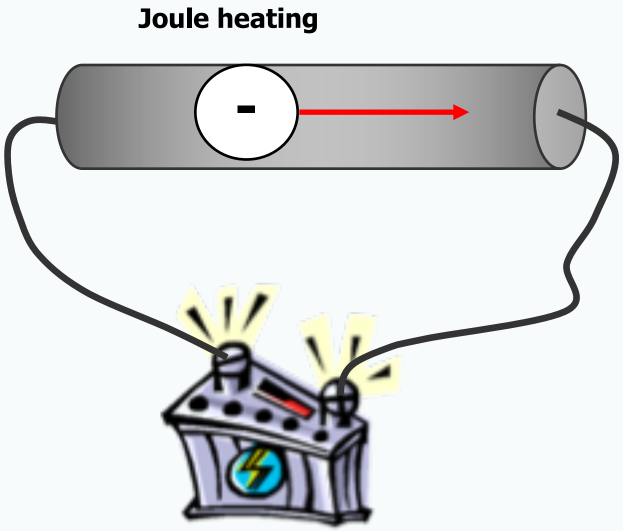


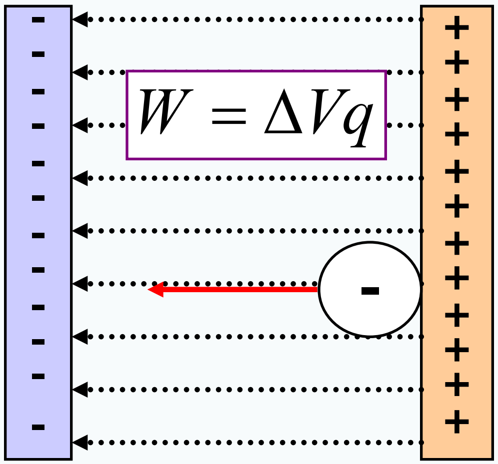


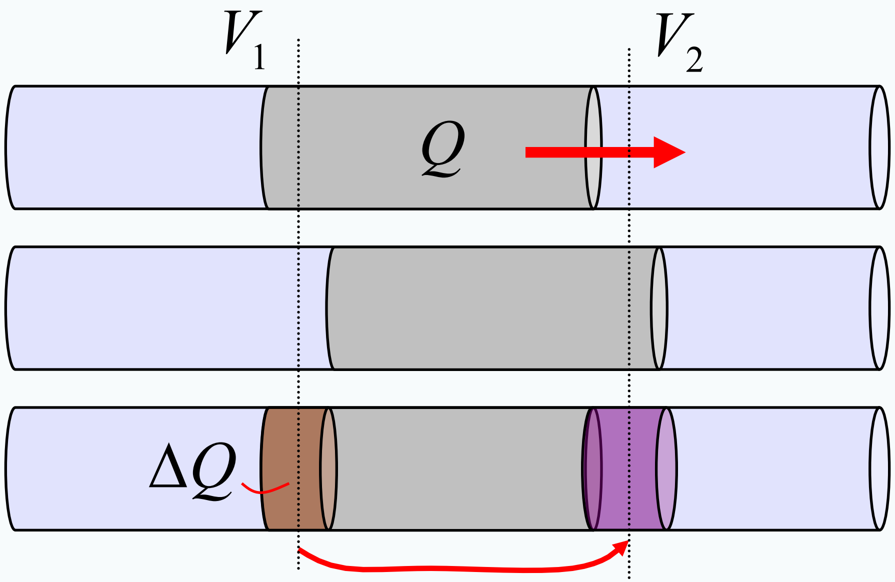




Power





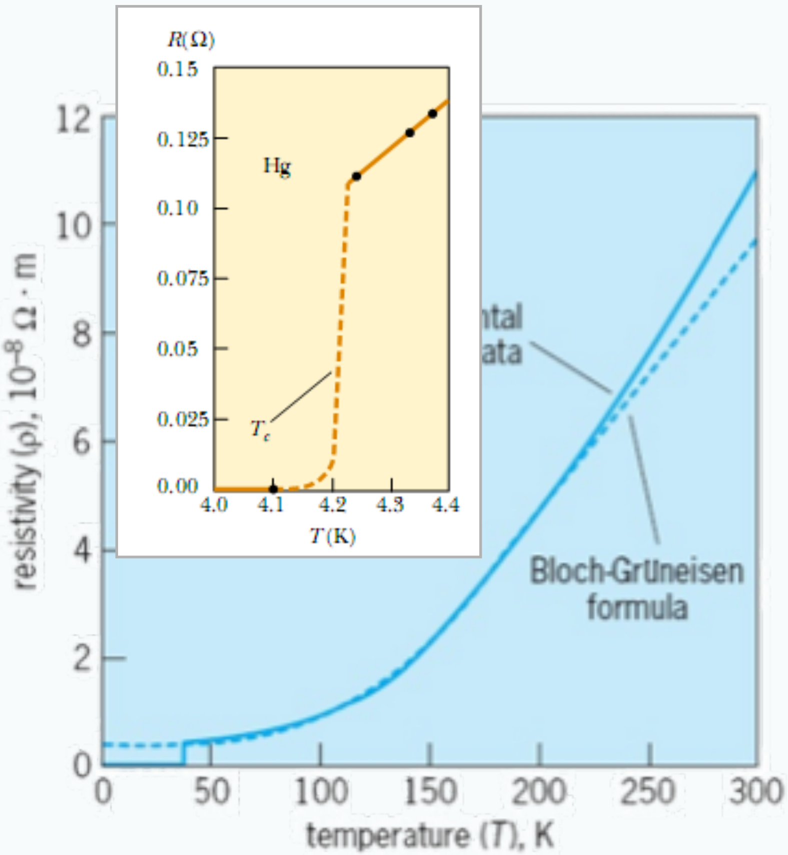


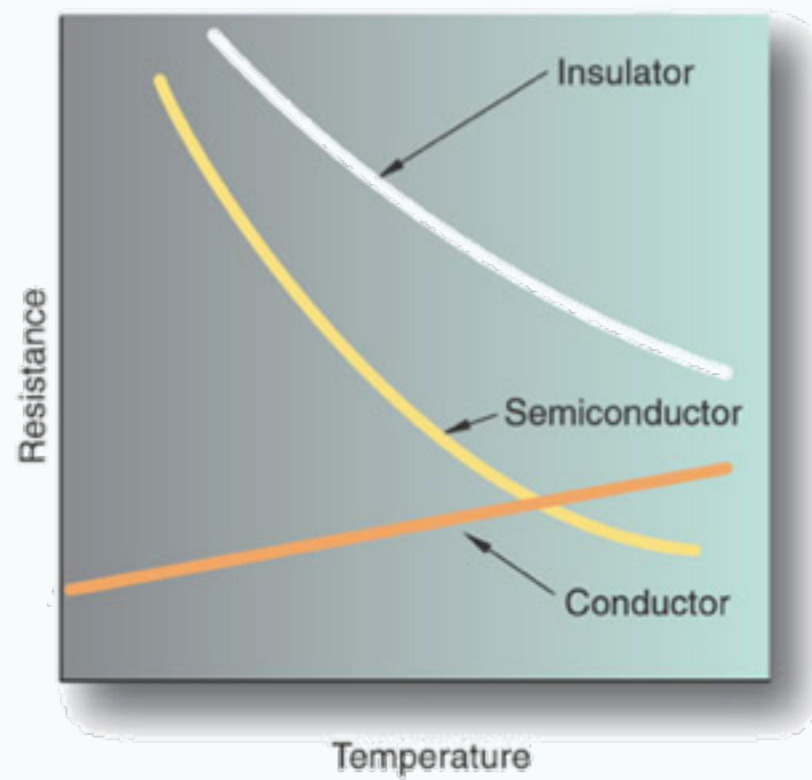
$$\frac{\Delta U}{\Delta t} = \frac{\Delta V \Delta Q}{\Delta t} = I \Delta V$$

$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

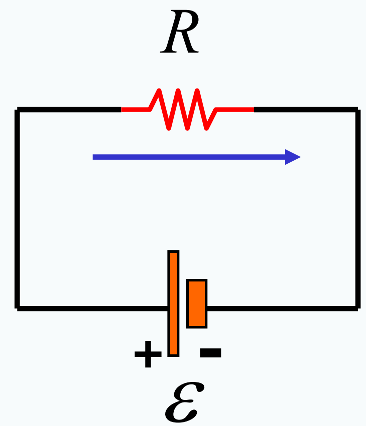
Temperature coefficient of resistivity

$$\boldsymbol{R = \rho \frac{l}{A}} \qquad \rho = \rho_0\left(1 + \alpha\left(T - T_0\right)\right) \qquad \alpha = \frac{1}{\rho_0}\frac{d\rho}{dT}$$

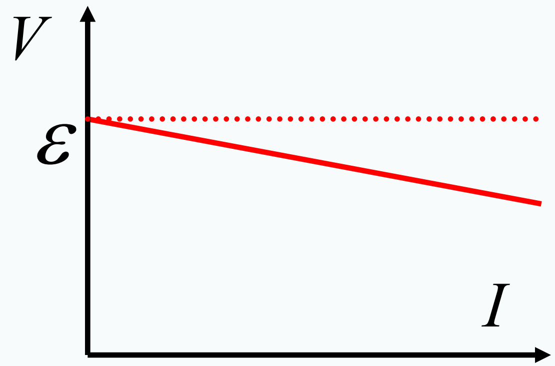




Electromotive force



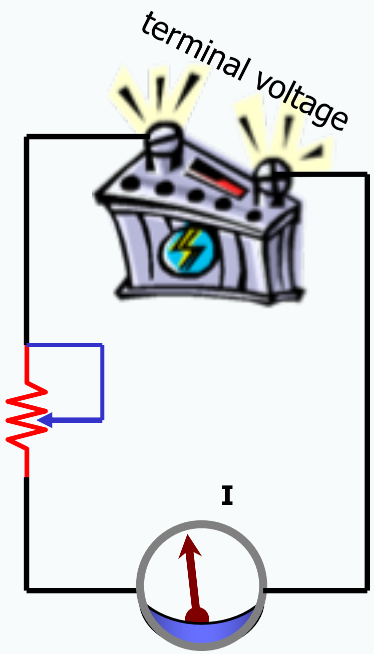
The work per unit charge is called the *emf* of the source.

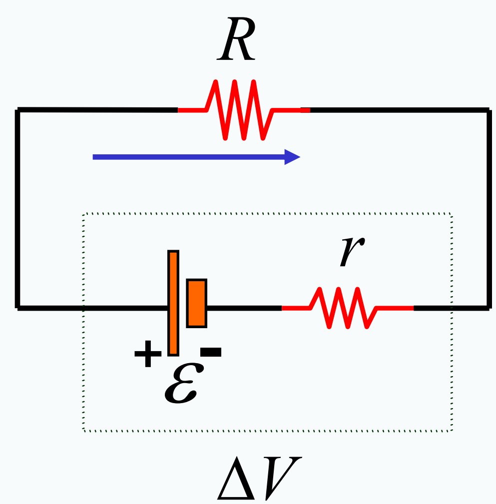


$$\Delta V = \mathcal{E} - Ir$$

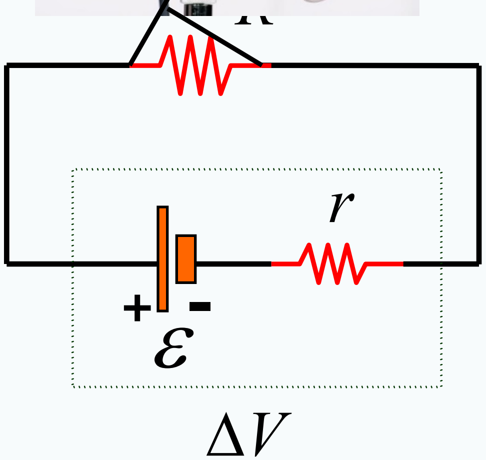
$$\Delta V = IR$$

$$I = \frac{\mathcal{E}}{R + r}$$





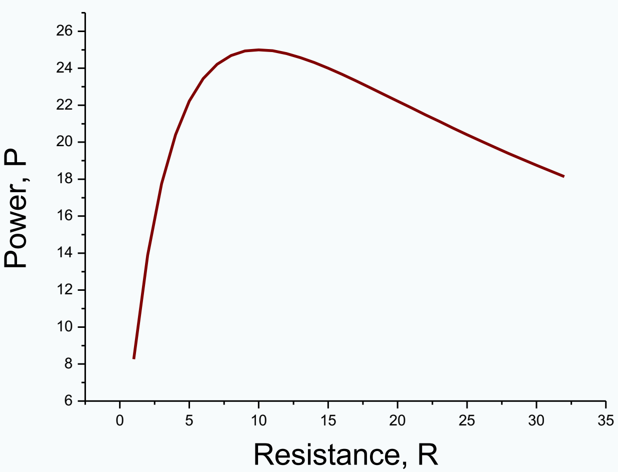
Maximal power delivered



$$I = \frac{\mathcal{E}}{R+r}$$

$$P = I^2 R = \frac{\mathcal{E}^2 R}{(R+r)^2}$$

$$\frac{dP}{dR} = \frac{\mathcal{E}^2}{(R+r)^2} - 2\frac{\mathcal{E}^2 R}{(R+r)^3} = 0$$

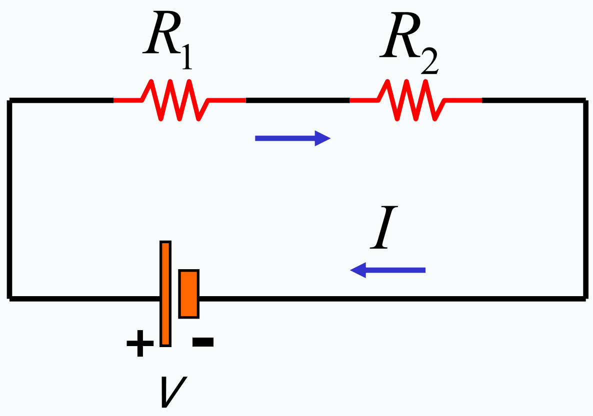


$$\boxed{R = r}$$

Impedance matching

Choosing $R = r$ to maximize the power delivered to the load resistor.

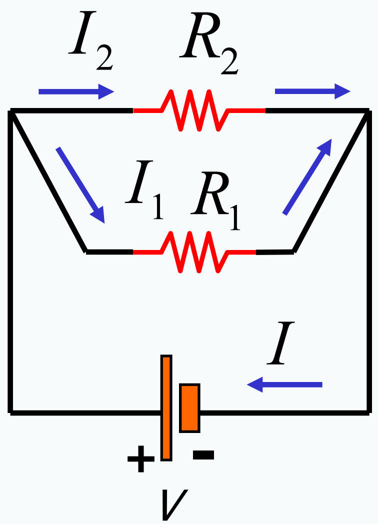
Combination of resistors



$$V = V_1 + V_2 = IR_1 + IR_2$$

$$R = R_1 + R_2$$

$$\boxed{R_{ser} = \sum R_i}$$



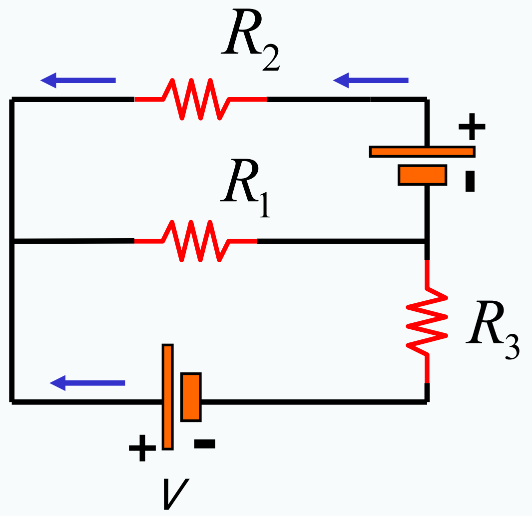
$$I = I_1 + I_2 \qquad V = I_1 R_1 = I_2 R_2$$

$$\frac{1}{R} = \frac{I}{V} = \frac{I_1 + I_2}{V} = \frac{V/R_1 + V/R_2}{V}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

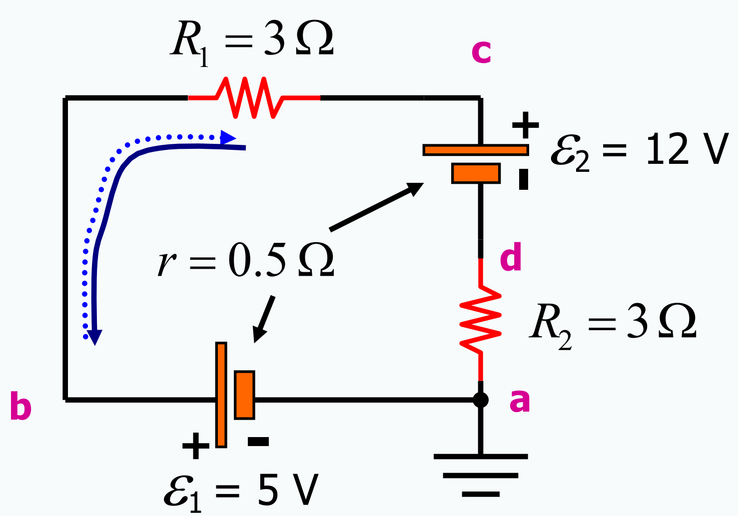
$$\boxed{R_{\parallel}^{-1} = \sum R_i^{-1}}$$

Kirchhoff's rules



When any closed loop is considered, the sum of the changes in electric potential must be zero.

At any branching point the sum of all currents (incoming and outcoming) must be zero.



$$\mathcal{E}_1 - Ir_1 - IR_1 - \mathcal{E}_2 - Ir_2 - IR_2 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + r_1 + r_2} = -1\ \text{A}$$

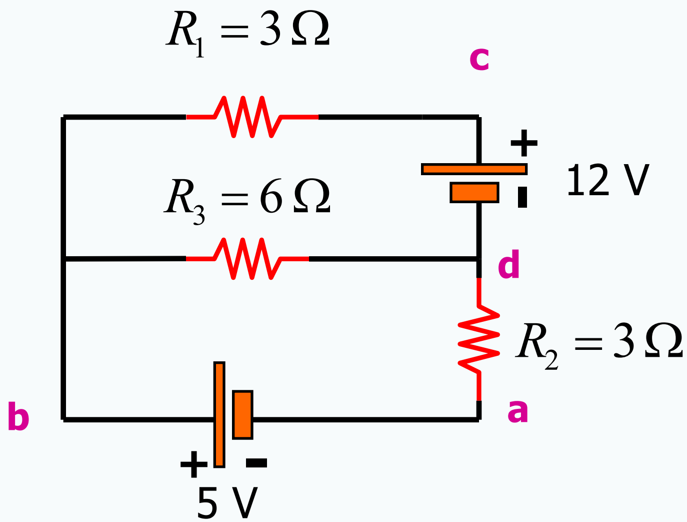
$$V_a = 0\,V \qquad V_d = V_a - 1\cdot 3 = -3\ \text{V}$$

$$V_c = V_d + 12 - 1\cdot 0.5 = 8.5\ \text{V}$$

$$V_b = V_c - 1\cdot 3 = 5.5\ \text{V}$$

$$V_a = V_b - 5 - 1\cdot 0.5 = 0\ \text{V}$$

Kirchhoff's rules



$$\boxed{I_1 + I_2 - I_3 = 0}$$

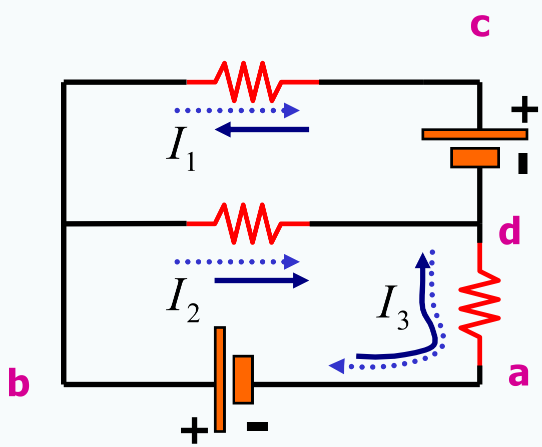
$$-I_1R_1 - 12\text{ V} + I_2R_3 = 0 \qquad /3\Omega$$

$$\boxed{-I_1 - 4\text{ A} + 2I_2 = 0}$$

$$-I_2R_3 - I_3R_2 + 5\text{ V} = 0$$

$$\boxed{-6I_2 + 5\text{ A} - 3I_3 = 0}$$

$$\boxed{I_1 = -\frac{26}{15}\text{ A}} \quad \boxed{I_2 = \frac{17}{15}\text{ A}} \quad \boxed{I_3 = -\frac{9}{15}\text{ A}}$$

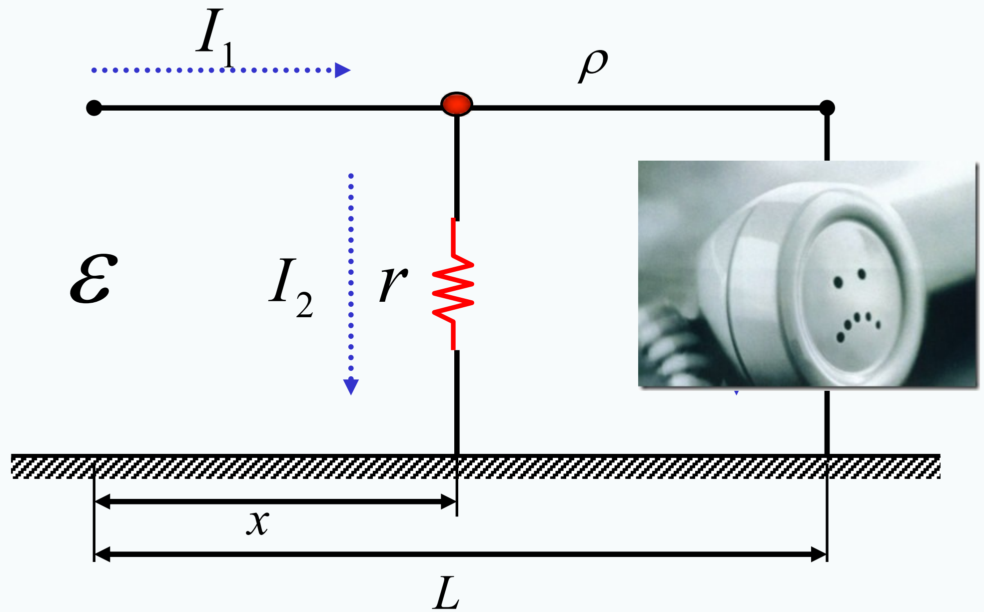


$$P_{R_2} = I_3^2R_2 = \frac{81}{225}\cdot 3 = 1.08\text{ W} \qquad W_{R_2} = P\Delta t = 5.4\text{ J}$$

$$P_{R_3} = 7.71\text{ W} \qquad P_{R_1} = 9.01\text{ W} \qquad \sum P_R = 17.8\text{ W}$$

$$P_{\varepsilon_{12}} = I_1\cdot\varepsilon_{12} = 20.8\text{ W} \qquad P_{\varepsilon_5} = I_3\cdot\varepsilon_5 = 3\text{ W}$$

Real life story



$$I_1 = I_2 + I_3$$

$$\mathcal{E} - x\rho I_1 - I_2 r = 0$$

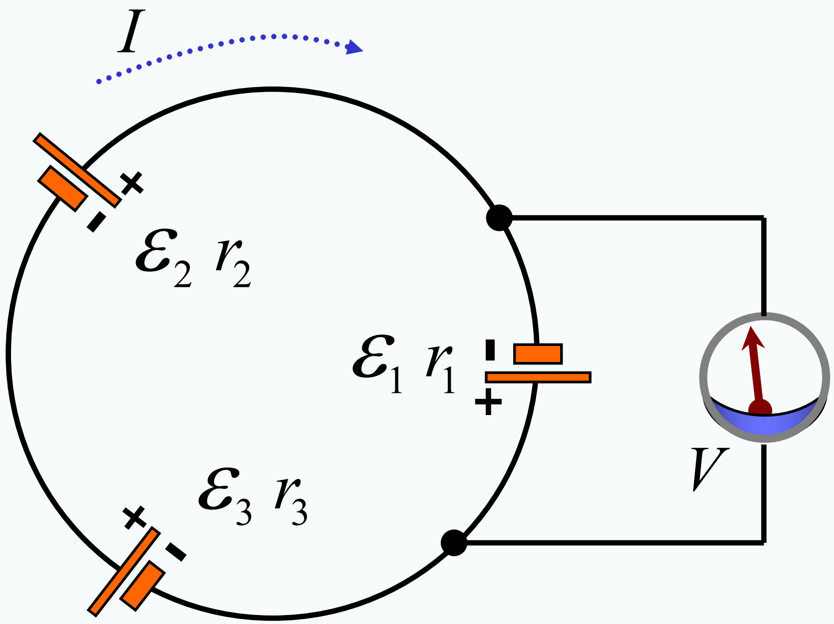
$$I_2 r - (L - x)\rho I_3 = 0$$

$$\mathcal{E} = \left[x\rho + \frac{x\rho r}{(L-x)\rho} + r \right] I_2$$

$$I_3 = \frac{\mathcal{E} r}{\rho\left(Lx\rho - x^2\rho - rL\right)}$$

$$\boxed{x = \frac{L}{2}}$$

Simple circuits



$$r_1 = r_2 = r_3 = 1\,\Omega$$

$$\mathcal{E}_1 = 11\,V \qquad \mathcal{E}_2 = \mathcal{E}_3 = 5\,V$$

$$\mathcal{E}_1 - Ir_1 + \mathcal{E}_2 - Ir_2 + \mathcal{E}_3 - Ir_3 = 0$$

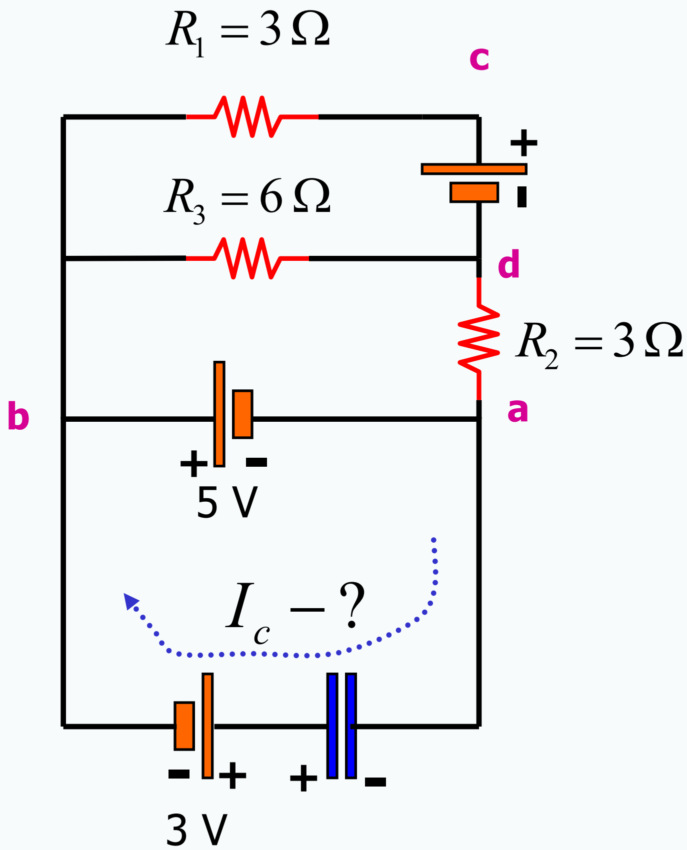
$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3}{r_1 + r_2 + r_3} = 7\ \mathrm{A}$$

$$\Delta V_1 = -\varepsilon_1 + Ir_1 = -4\ \mathrm{V}$$

$$\Delta V_2 = -\varepsilon_2 + Ir_2 = 2\ \mathrm{V} \qquad \Delta V_3 = -\varepsilon_3 + Ir_3 = 2\ \mathrm{V}$$

$$\mathcal{E} = K \cdot r \qquad \Delta V_i = -\mathcal{E}_i + Ir_i = -\mathcal{E}_i + r_i \frac{K\sum r_i}{\sum r_i} = 0$$

Adding capacitor – steady state



Adding an open circuit does not affect currents in other parts of the circuit.

$$+\Delta V_c - 3\text{ V} - 5\text{ V} = 0$$

$$\Delta V_c = 8\text{ V}$$

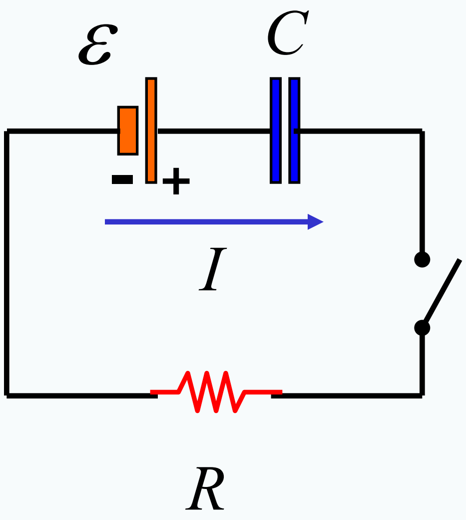
$$Q = C\Delta V_c = 1\,\mu\text{F}\cdot 8\text{ V} = 8\,\mu\text{C}$$

$$-\Delta V_c + 3\text{ V} - 5\text{ V} = 0$$

$$Q = 2\,\mu\text{C}$$

Polarity -? $Q - ?$

Charging capacitor



$$\mathcal{E} - \frac{q}{C} - IR = 0$$

$$t = 0: \qquad \mathcal{E} - IR = 0$$

$$t = \infty: \qquad \mathcal{E} - \frac{Q}{C} = 0$$

$$\boxed{\mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = 0}$$

$$q(t) = \mathcal{E}C\left(1 - \exp\left\{-\frac{t}{RC}\right\}\right) \qquad I(t) = \frac{\mathcal{E}}{R}\exp\left\{-\frac{t}{RC}\right\}$$

Discharge:

$$q(t) = \mathcal{E}C\exp\left\{-\frac{t}{RC}\right\} \qquad I(t) = -\frac{\mathcal{E}}{R}\exp\left\{-\frac{t}{RC}\right\}$$

To remember!

- Electric current is charge passing through a given area per unit time.
- Current density in a conductor is proportional to the electric field.
- If a potential difference is maintained across a resistor, the power supplied to it will be equal to the product of the potential difference and the current.
- Real battery can be considered as an ideal one connected in series with a small resistance called internal resistance.
- For serial connection the resistance are summed; for parallel - their reciprocal values.
- When a closed-circuit loop is traversed, the sum in the changes of potential must be zero.
- At any junction the sum of inflowing current must be equal to the sum of currents flowing out.

To remember!

- When a closed-circuit loop is traversed, the sum in the changes of the electric potential must be zero.
- At any junction the sum of the inflowing currents must be equal to the sum of the currents flowing out.
- No current flows through an open circuit, e.g. through capacitor.
- In non-steady state, however, transient currents can exist, e.g. during charging and discharging capacitors.
- The product RC is called the time constant of a circuit. It defining, e.g., rate of exponential lost of the charge by capacitor during its discharge.