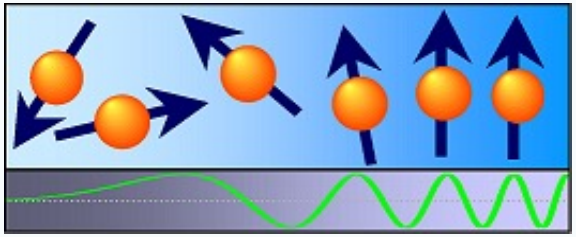
Experimental Physics EP2
Thermodynamics

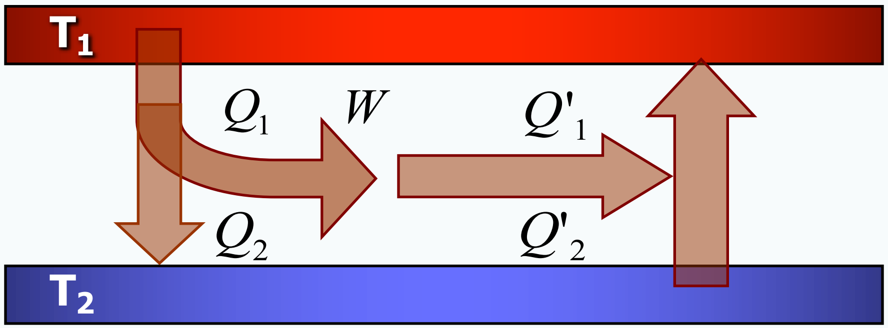
– Entropy –

Carnot cycle, Clausius inequality



https://bloch.physgeo.uni-leipzig.de/amr/

The Clausius inequality



$$\frac{T_2}{T_1} = \frac{Q'_2}{|Q'_1|}$$

$$\frac{Q'_1}{T_1} + \frac{Q'_2}{T_2} = 0$$

$$W = Q_1 + Q_2 + Q'_1 + Q'_2 = 0 \quad \Rightarrow \quad Q_1 + Q'_1 = -(Q_2 + Q'_2)$$

$$0 = Q_1 + Q_2 + Q'_1 - Q'_1 \frac{T_2}{T_1} \quad \Rightarrow \quad Q'_1 = T_1 \frac{Q_1 + Q_2}{T_2 - T_1}$$

$$Q_1 + Q'_1 = \frac{T_1 T_2}{T_2 - T_1}\left[\frac{Q_1}{T_1} + \frac{Q_2}{T_2}\right]$$

$$if\ T_1 > T_2\ then\ Q_1 + Q'_1 \geq 0$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \leq 0$$

System:

$$\frac{Q_1}{T_1} - \frac{|Q_2|}{T_2} \leq 0$$

$$1 - \frac{|Q_2|}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

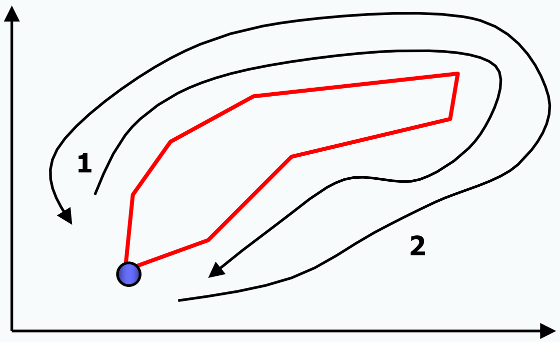
$$\eta_{engine} \leq \eta_{ideal}$$

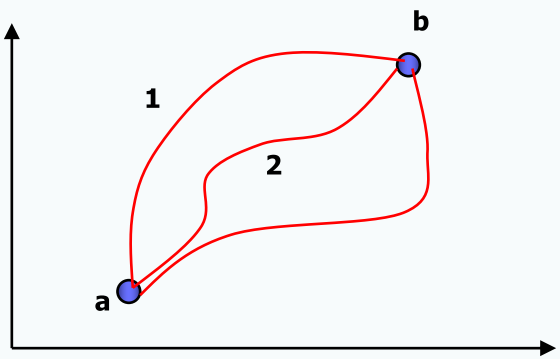
Entropy

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \le 0$$

$$\oint \frac{\delta Q}{T} \le 0$$

Quasistatic reversible process





1. $$\oint \frac{\delta Q}{T} \le 0$$

2. $$\oint \frac{-\delta Q}{T} \le 0$$

$$\oint_{\substack{quasi\\static}} \frac{\delta Q}{T} = 0$$

$$\int_{a1b} \frac{\delta Q}{T} + \int_{b2a} \frac{\delta Q}{T} = 0 \qquad \int_{a1b} \frac{\delta Q}{T} - \int_{a2b} \frac{\delta Q}{T} = 0$$

$$\int_{a1b} \frac{\delta Q}{T} = \int_{a2b} \frac{\delta Q}{T} = \ldots \int_{aNb} \frac{\delta Q}{T}$$

$$S_b - S_a = \int_a^b \frac{\delta Q}{T}$$

$$dS = \frac{\delta Q}{T}$$

entropy

Entropy of ideal gas

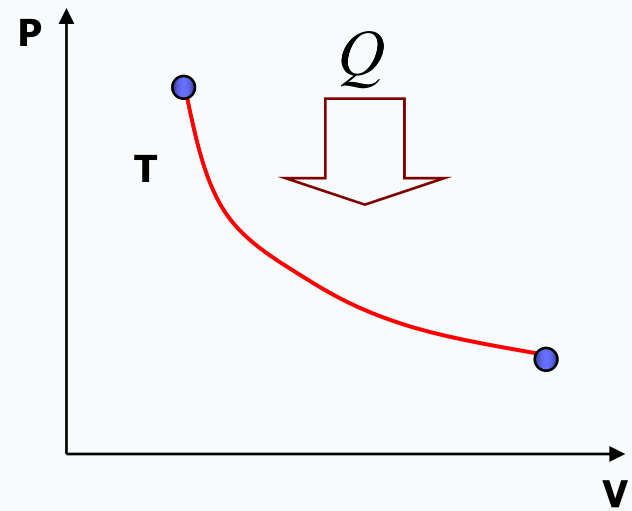
$$\delta Q = C_v dT + PdV = C_v dT + \nu RT \frac{dV}{V}$$

$$dS = \frac{\delta Q}{T} = C_v \frac{dT}{T} + \nu R \frac{dV}{V}$$

$$S = C_v \ln(T) + \nu R \ln(V) + const$$

$$\Delta S = C_v \ln\left(\frac{T_2}{T_1}\right) + \nu R \ln\left(\frac{V_2}{V_1}\right)$$

Isothermal process:

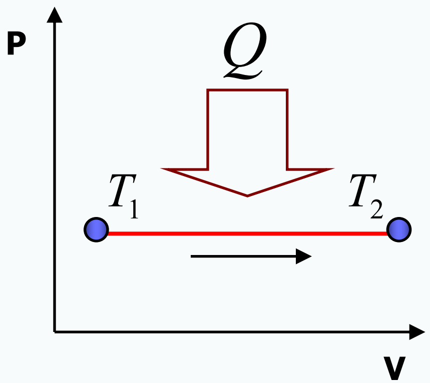


$$\Delta S = \nu R \ln\left(\frac{V_2}{V_1}\right) > 0$$

$$\Delta S_{gas} = \frac{|Q|}{T} \qquad \Delta S_{reservoir} = \frac{-|Q|}{T}$$

For reversible processes:

$$\Delta S_{universe} = 0$$

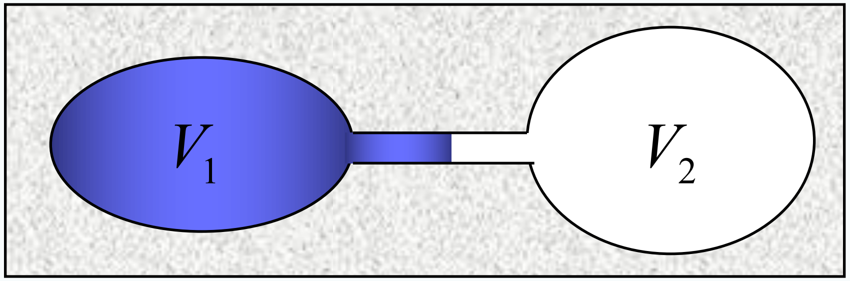


Isobaric process:

$$dS = \frac{\delta Q}{T} = C_p \frac{dT}{T}$$

$$\Delta S = C_p \ln\left(\frac{T_2}{T_1}\right)$$

Entropy of ideal gas: free expansion



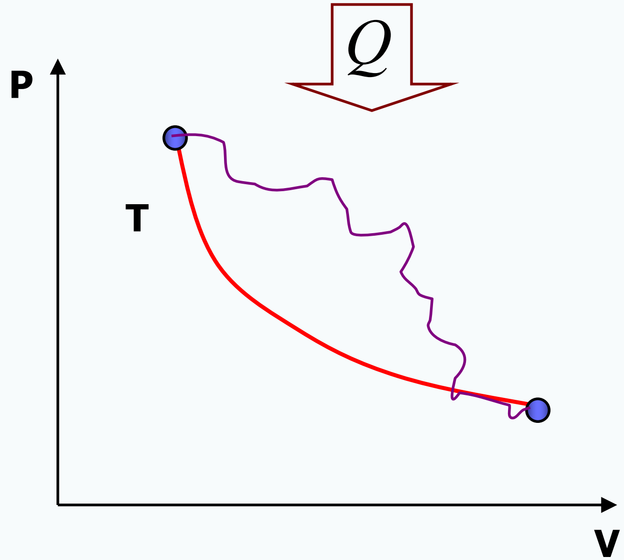
$$dS = \frac{\delta Q}{T}$$

T = const

δQ = 0

$$\Delta S = 0$$

Not directly applicable for irreversible processes!

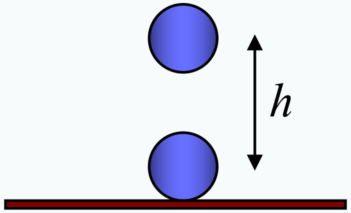


$$\Delta S = \nu R \ln\left(\frac{V_1 + V_2}{V_1}\right)$$

In an irreversible process the entropy of universe increases.

In a reversible process the entropy of universe does not change.

For any processes, the entropy of universe does not decrease.

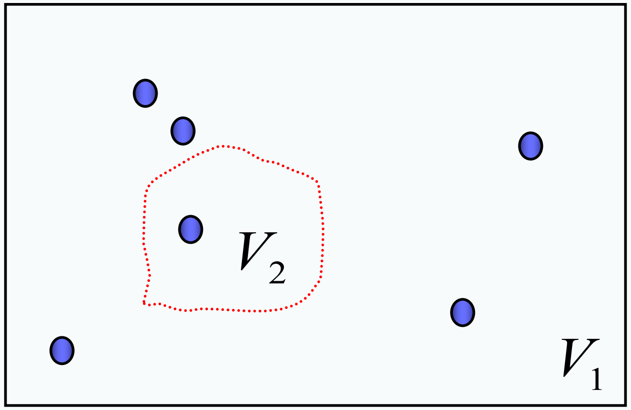


$$\Delta S = \frac{mgh}{T}$$

Energy unavailable for work

$$T\Delta S$$

Entropy and probability



$$V_1 = 2V_2 \Rightarrow T\Delta S = \nu RT\ln(2)$$

$$p = \frac{V_2}{V_1}\cdot\frac{V_2}{V_1}\cdots\frac{V_2}{V_1} = \left(\frac{V_2}{V_1}\right)^N$$

$$\ln(p) = N\ln\left(\frac{V_2}{V_1}\right) = \nu N_A \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S = \nu R\ln\left(\frac{V_1}{V_2}\right)$$

$$\Delta S = k\ln(p^{-1}) \Rightarrow S = k\ln(\Omega)$$

High order → low order
Less probable → more probable

To remember!

- **The efficiency of a Carnot engine is determined only by temperature difference between hot and cold reservoirs.**
- **The efficiency of any engine cannot exceed that of the respective ideal engine, i.e., the Carnot engine.**
- **Heat added to a system via a quasistatic reversible process and normalized to temperature is equal to entropy change.**
- **In a closed system entropy never decreases: For reversible processes it remains constant, for irreversible processes it increases.**
- **Entropy is a measure of disorder.**